Unless there are unsuspected sources of error, this value of a for air should not be more than 15 per cent. from the true value.

Summary.

It has been found experimentally that a cathode ray moving with velocity v_0 can possess, after traversing a thickness x of material, a velocity v_x given by the relation

$$v_0^4 - v_x^4 = ax,$$

where a is a constant depending on the nature of the material.

This constant a has been measured for Al, Au, and air at 760 mm. pressure of mercury and 15° C., with the following results:—

Al	$7 \cdot 32 \times 10^{42}$
Au	$2 \cdot 54 \times 10^{43}$
Air	$2 \cdot 0 \times 10^{40}$

There seems to be no simple law connecting the value of a with the density or atomic weight of the absorbing material.

The Velocity of the Secondary Cathode Particles Ejected by the Characteristic Röntgen Rays.

By R. Whiddington, M.A., Fellow of St. John's College, Cambridge.

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Introduction.

When a metal plate is "illuminated" by Röntgen rays, part of the energy of these incident rays is converted into high-speed secondary cathode particles, which are ejected from the plate in all directions. The mere detection of this emission of negative electricity is an easy matter, since the illuminated plate, if insulated *in vacuo*, charges up positively. By measuring the rate of charging, it would be possible to determine the number of particles ejected per second, while by applying an electric force of such a magnitude and in such a direction as to stop the emission, it might be thought that their speed could be measured. In practice, however, this method of measuring velocities is restricted to slowly moving rays, such as are produced, for example, by ultra-violet light. The particles ejected by

Röntgen rays have in most cases a speed corresponding to a potential fall through many thousands of volts, and at present it is not possible to perform accurate experiments with potentials of this magnitude. Quite the best way of determining high velocities is the magnetic deflection method, in which a magnetic force of known strength is applied to a fine beam of the moving particles in such a way as to act at right angles to their direction of motion; from observations of the curvature of the path described by the particles their speed can be calculated. This method, unfortunately, is at present impossible in the case of secondary cathode particles, since their intensity is never sufficient to allow of their being formed into anything like a narrow beam.

There is, however, a method of attacking the problem which it is the object of this paper to outline. The experimental data available are somewhat incomplete, and the conclusions drawn from them of necessity share this incompleteness.

The method is based on the fact that the secondary cathode particles ionise the gas they happen to be traversing, an effect which may be quite large with a small number of particles, since each particle is capable of producing very many ions before being brought to rest.

Now it is known that the distance d which a cathode particle can travel through a gas depends on its initial velocity v_s, so that if the exact form of the relation

$$d = f(v_s)$$

were known, it would be possible to calculate at once the velocity of a cathode particle whose range was known. I have attempted to work out this idea, using some experiments of Beatty and of Sadler to furnish values for d, and the experiments of the preceding paper to indicate the form of the relation $d = f(v_s)$.

The terminology introduced by Prof. Bragg will be used. For example, the Röntgen radiation characteristic of Sn will be referred to as the Sn X-rays, while the cathode particles ejected by Sn X-rays will be termed Sn cathode rays.

Beatty and Sadler have, independently, and by slightly different methods, investigated the variation of the ionisation produced by a stream of cathode particles as the thickness or effective thickness of the stratum of air they traversed was altered.

Sadler* actually altered the depth of his ionisation chamber by using a movable plane collecting electrode. Beatty† adopted the superior plan of

* Sadler, 'Phil. Mag.,' March, 1910.

† Beatty, 'Phil. Mag.,' August, 1910.

altering the *effective depth* by varying the pressure of the gas between two parallel electrodes a fixed distance apart. The effective depth (referred to in future merely as the depth) is proportional to the pressure.

In both methods parallel plate electrodes are supported opposite each other with a saturating electric field set up between them. The high potential plate is "illuminated" with a characteristic Röntgen radiation, and consequently a stream of cathode particles proceeds towards the other insulated electrode, ionising the intervening gas. It is this ionisation which is measured.*

The type of curve which is obtained by plotting I, the ionisation, against x, the depth of the chamber, is shown in fig. 1.

When x is greater than d, none of the particles reach the opposite plate, so that, increasing x above this value d does not increase I, which assumes a steady value, depending on the intensity and type of the exciting rays.

Beatty's results will be referred to most frequently, so it is convenient to point out here that what he actually tabulates is t, the distance between his plates necessary (at 760 mm. pressure of mercury and 15° C.) to give half the greatest possible ionisation, and a quantity λ proportional to $1/t$.

It must be possible to represent the experimental curve of fig. 1 by an expression containing terms which take into account, firstly, the fact that the velocity of the cathode particles is continuously diminishing as the distance traversed increases, and, secondly, the fact that some of the particles may get reduced to rest (by becoming permanently lodged within a molecule or in some such way), thus converting their energy suddenly into heat instead of completing their ionising career.

Such terms would be $$v_s^4 - v_x^4 = ax, \dagger \tag{1}$$

where v_s is the velocity of a particle at the moment of projection from the plate, v_x is the velocity after traversing distance x, a is some constant depending on the nature of the gas; and

$$N = N_0 e^{-\gamma x}, \tag{2}$$

where N_0 is the number of particles emitted per second by unit area of the illuminated plate, N is the number crossing unit area a distance x away, γ is some function of x.

It appears that the expression

$$I = k_1 \int_0^x e^{-\gamma x} (v_s^4 - ax)^{\frac{1}{2}} \, dx \tag{3}$$

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* For the method of allowing for that part of the ionisation due to the Röntgen rays, reference must be made to the original papers.

† See preceding paper on cathode ray transmission.

represents Beatty's curves extremely well. The constant k_1 depends on the initial number of the cathode particles.

Relation (3) may be regarded as empirical, since the hypotheses on which its deduction depends seem unjustifiably simple.

It follows at once if we assume—

(1) That all the rays starting from the illuminated plate have the same velocity v_s, which cannot be the case, since the ejected particles must, some of them, have started from below the surface.

(2) That the ionisation produced in a thin layer of gas is proportional to the energy of the travelling particles, an hypothesis not in accord with some experiments carried out by Glasson,* from which it follows that, at any rate, over part of the range here considered, the ionisation is proportional to the *inverse* of the energy.

(3) That γ is a constant, an obviously improbable assumption.†

The fact remains, however, that (3) fits the experimental results very well, and it seems an interesting illustration of the possibility of suitably combining erroneous hypotheses so as to arrive finally at a correct formula.

(3) as it stands is not in a form suited to evaluation, but by making the substitution

$$v_s{}^4 - ax = a/\gamma \, . \, z^2$$

it assumes the form

$$\mathrm{I} = k_1 (a/\gamma^3)^{\frac{1}{2}} e^{-v_s{}^4 \gamma/a} \left\{ \int e_z{}^2 dz - z e^{z^2} \right\},$$

which lends itself readily to evaluation, since e^{z^2} can be obtained from mathematical tables, while $\int e^{z^2} . \, dz$ has been tabulated by Dawson.‡

In order to compare the form of the curve represented by (3) with that obtained experimentally, it is necessary to take a special value of v_s. It is convenient to take the case of the Zn cathode rays, the velocity of which will be shown later on to be $6 \cdot 3 \times 10^9$ cm./sec. A suitable value for γ turns out to be 29.

The constants in (3) become therefore

$$v_s = 6 \cdot 3 \times 10^9, \qquad a = 2 \times 10^{40}, \qquad \gamma = 29.$$

Now $e^{-v_s{}^4 \gamma/a}$ is practically constant except very near the origin of the curve, so that to simplify the calculation one can so choose k_1 as to make

$$k_1 (a/\gamma^3)^{\frac{1}{2}} e^{-v_s{}^4 \gamma/a} = 1, \quad \text{in which case} \quad \mathrm{I} = \int e^{z^2} dz - z \, . \, e^{z^2}.$$

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* Glasson, 'Phil. Mag.,' November, 1911.

† Without making this assumption, (3) would not lend itself to ready transformation and evaluation. It will be seen, however, that γ is absent from the formulæ, (4) and (6), expressing the main results of this paper.

‡ Dawson, 'Lond. Math. Soc. Proc.,' 1898, vol. 29, p. 519.

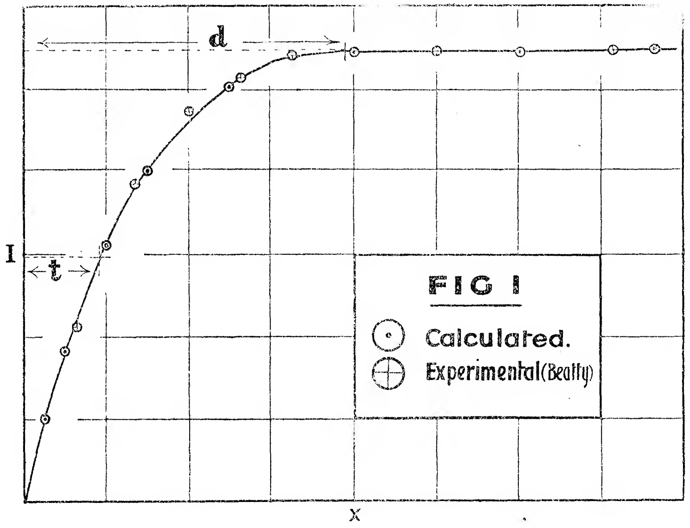
The following table shows the dependence of I on x, using only positive values of z. The expression (3) has its greatest (real) value when $z = 0$, *i.e.* when $x = v_s^4/a = 0{\cdot}079$ cm.

The numerical value of the constant of integration C is found by putting $x = 0$ when $I = 0$.

Table I.

x.	z^2.	z.	$\int e^{z^2}dz$.	ze^{z^2}.	$I = \int e^{z^2}dz - ze^{z^2} + C$.
0·079	0·000	0·000	0·000	0·000	10·941
0·05	0·836	0·915	1·250	2·120	10·071
0·03	1·421	1·192	2·108	4·950	8·099
0·02	1·711	1·310	2·675	7·324	6·294
0·0164	**1·815**	**1·348**	**2·977**	**8·390**	**5·530**
0·01	2·001	1·416	3·357	10·600	3·698
0·005	2·146	1·465	3·751	12·620	2·072
0·000	2·291	1·515	4·209	15·150	0·000

The results of this table are shown graphically in fig. 1, the points being marked thus ⊙ ; those indicated by ⊕ are the experimental points of Beatty for the Zn cathode rays.



As a further test of the accuracy of formula (3) in expressing the experimental results, the value of $x = t = 0{\cdot}0164$ cm. given by Beatty is included in the above table. The ionisation I is seen to be very nearly equal to half the maximum, differing from it by only 1 per cent.

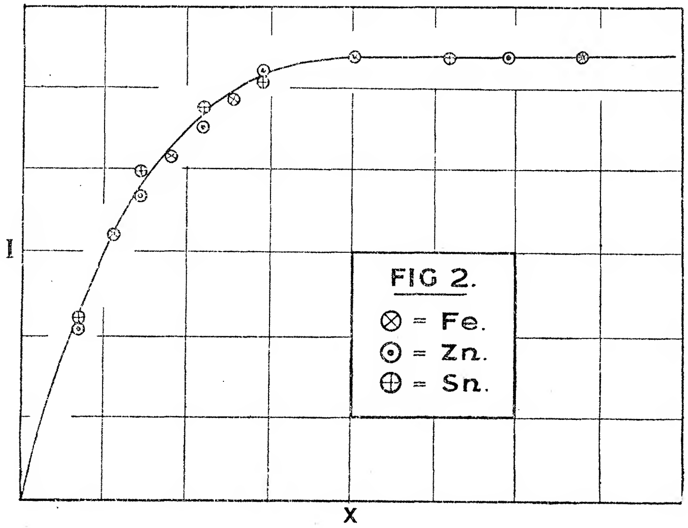
It follows from (3) that the greatest range d of a cathode particle of initial speed v_s is the value of x given by putting

$$v_s^4 - ax = 0, \quad \text{whence} \quad d = v_s^4/a. \qquad (4)$$

So that d is independent of γ.

From this very simple expression it is possible from a knowledge of the range of a cathode ray to calculate the initial velocity.

It has already been pointed out that Beatty and Sadler tabulate a quantity λ which is proportional to $1/t$, t being the thickness of air necessary to reduce the observed ionisation to one-half its maximum value. It is essential, therefore, to get the connection between d and t.



In fig. 2 the experimental curves (Beatty) for the Fe, Zn, and Sn cathode rays are plotted together on the same scale. It is obvious that all the points lie very close to a common curve.

It is, therefore, legitimate to assume that

$$d \propto t \propto 1/\lambda. \tag{5}$$

Combining (4) and (5), we have

$$\lambda v_s^4 = \text{constant}, \tag{6}$$

a relation which affords a ready means of comparing the velocities of the various types of cathode ray.

Application of Relation (6) *to the Experiments of Beatty and Sadler.*

In the following table are collected, for purposes of comparison, the results of Beatty and Sadler. The numbers given are in all cases the values of λ for the various types of cathode ray ranging from the easily absorbed Fe cathode ray to the penetrating Sn cathode ray:—

Table II.

Tertiary radiator.	Secondary radiator.								
	Fe.	Cu.	Zn.	As.	Se.	Sr.	Mo.	Ag.	Sn.
Iron (S.) ……………	—	37·0	35·8	30·2	26·4	21·5	**15·8**	**8·84**	**6·41**
Silver (B.) …………	87·2	51·9	42·7	27·4	—	—	—	—	3·97
Selenium (B.)………	—	—	—	—	—	16·1	**14·0**	**8·8**	**6·5**

This table shows that in only three cases are the two observers in really good agreement, viz., for the Mo, Ag, and Sn cathode rays.

Table III shows that for these three types of ray the relation

$$\lambda w^4 = \text{constant} \tag{7}$$

is true to an accuracy of a few per cent., w being the atomic weight of the radiator supplying the characteristic Röntgen rays:—

Table III.

Secondary radiator.	λ (B.).	λw^4.
Mo	14·0	$1·19 \times 10^9$
Ag	8·8	$1·20 \times 10^9$
Sn	6·5	$1·16 \times 10^9$

It is very unfortunate that the data for the radiators of atomic weight less than that of Mo should be conflicting. It may be noticed, however, that (7) is true for all Beatty's numbers, while it applies to Sadler's only over the limited range Mo—Sn.*

Table IV shows the constancy of λw^4 for the whole of Beatty's range:—

Table IV.

Secondary radiator.	λ (B.).	λw^4.
Fe	87·2	$0·85 \times 10^9$
Cu	51·9	$0·85 \times 10^9$
Zn	42·7	$0·78 \times 10^9$
As	27·4	$0·86 \times 10^9$
Sn	3·97	$0·78 \times 10^9$

Combining (6) and (7), we arrive at the conclusion that

$$v_s = k'w. \tag{8}$$

That is, *the velocity of the cathode rays ejected by a characteristic Röntgen radiation is proportional to the atomic weight of the radiator supplying the Röntgen rays.*

It yet remains to find the value of k'.

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* An examination of Sadler's methods shows that the limitations of his apparatus necessitated the use of a mixture of hydrogen and air for the slower cathode rays, deducing from a density law (assumed) the values of λ for air from those obtained using this mixture. Beatty's results for these slower rays are therefore more trustworthy.

An estimate of its value can be obtained from an examination of Beatty's curves. It is possible to get from them an estimate of d, the range for the Fe, Zn, and Sn cathode rays (the only three for which curves are given), to an accuracy of about 2 per cent. By substituting these values of d in (4) we get the values of v_s given in Table V. From (4) we see that

$$v_s = (ad)^{\frac{1}{4}}.$$

From which it follows that the error in v_s should be not more than about 4 per cent. :—*

Table V.

Characteristic radiator.	d, in cm.	w.	v_s, in cm./sec.
Fe	0·0418	55·9	$5{\cdot}37 \times 10^9$
Zn	0·0790	65·3	$6{\cdot}30 \times 10^9$
Sn	0·850	119·0	$1{\cdot}14 \times 10^{10}$

The constant k' in (8) is thus approximately 10^8.

About a year ago I found an experimental law of the same form as (8), as a result of some work on the production of characteristic Röntgen radiations.† The law referred to can be expressed thus :—

$$v_p = kw. \tag{9}$$

In this formula v_p is the velocity of those primary cathode rays which, striking an anticathode, emit Röntgen rays just sufficiently penetrating to excite the radiation characteristic of the element of atomic weight w. To put it in a slightly different way, the radiation characteristic of the element of atomic weight w is indistinguishable from the hardest constituent of the radiation emitted from a target struck by a stream of cathode rays of velocity equal to kw ; k is approximately equal to 10^8. Thus k and k' are very nearly equal, a point of the very greatest interest.

The points brought out in the preceding discussion can be conveniently summarised by following out the various energy transformations taking place between the arrest of the original parent cathode rays within a Röntgen bulb and the subsequent liberation of secondary cathode rays in a distant tertiary radiator by means of characteristic Röntgen rays.

Thus parent (primary) cathode rays of velocity just greater than $v_p = kw$ give rise to primary Röntgen rays, which give rise to secondary Röntgen rays characteristic of element of atomic weight w, which give rise to secondary cathode rays of velocity $v_s = k'w$.

Also, since $k = k'$, we must have, to the same order of accuracy, $v_s = v_p$.

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* Since from the preceding paper a is probably correct to within 15 per cent.

† Whiddington, 'Roy. Soc. Proc.,' A, vol. 85, p. 331.

[*Added February* 20.—Prof. Barkla has recently performed experiments which show that there is more than one characteristic radiation associated with some of the elements. His experiments clearly demonstrate the existence of at least two series of radiations, which he distinguishes by the letters K and L, the latter series being the new one. The present investigation has dealt only with the radiations of series K, but the results can be easily extended to include series L, as will now be shown.

It was pointed out by the writer* that Prof. Barkla's results could be conveniently summarised under one simple empirical formula expressing the fact that so far as Röntgen ray emission is concerned, an element of atomic weight w behaves as though it possessed a variable atomic weight $w' = \mathrm{A}w + \mathrm{B}$, where the values of A and B depend on the series to which the radiation belongs. Now if we define the quality of a Röntgen radiation in terms of the speed of the slowest cathode rays which can excite it, we can state that the radiations characteristic of the element of atomic weight w are indistinguishable from (if not the same as) the hardest constituents of the radiations emitted from a target struck by cathode rays of velocities given by

$$v_p = k(\mathrm{A}w + \mathrm{B}) \qquad (10)$$

when A and B are assigned suitable values.†

If we require the radiation of series K

$$\mathrm{A} = 1 \text{ and } \mathrm{B} = 0, \text{ so that (10) becomes (9).}$$

If we require the radiation of series L

$$\mathrm{A} = 1/2 \text{ and } \mathrm{B} = -25, \text{ so that } v_p = k(w/2 - 25).$$

In the same way the velocities of the secondary cathode particles ejected by these radiations can be obtained from the formula

$$v_s = k'(\mathrm{A}w - \mathrm{B}). \qquad (11)$$

By making the same substitutions for A and B in (11) as in (10), we conclude that if the radiations of both series, K and L, are stimulated in the element of atomic weight w, and the resulting mixture allowed to produce secondary cathode particles, there are two sets of particles produced possessing velocities

$$w \,.\, 10^8 \text{ cm./sec.} \quad \text{and} \quad (w/2 - 25)\, 10^8 \text{ cm./sec.}]$$

It gives me very great pleasure to thank Prof. Sir J. J. Thomson for his kind interest in these investigations.

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* 'Nature,' Nov., 1911, p. 143.

† See also 'Camb. Phil. Soc. Proc.,' vol. 16, part 4, p. 329.